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Discrete Lattice Systems and the Equivalence of Microcanonical, Canonical and Grand Canonical Gibbs States

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Abstract. It is proven that a microcanonical Gibbs measure on a classical discrete lattice system is a mixture of canonical Gibbs measures, provided the potential is "approximately periodic," has finite range and possesses a commensurability property. No periodicity is imposed on the measure. When the potential is not approximately periodic or does not have the commensurability property, the inclusion does not hold.

As a by-product, a new proof is given of the fact that for a large class of potentials, a canonical Gibbs measure is a mixture of grand canonical measures. Thus the equivalence of ensembles is obtained in the sense of identical correlation functions.

1. Introduction

There is a long standing tradition in statistical mechanics of modelling in terms of classical discrete lattice systems: lattice gases and spin systems (including the Ising, Ashkin–Teller, Potts and Z_n models) are amongst the most intensively studied objects in this research area [1]. A general and rigorous theory of their equilibrium properties and phase transitions was recently undertaken by Pirogov and Sinai [2, 3] and continued in [4] for instance. The mathematical context is that of a stochastic process taking values in some finite set, and indexed by the finite subsets of an infinite lattice. The underlying measure is a grand canonical Gibbs measure defined on the configuration space for the infinite system.

The present article is an attempt to justify this ansatz. Indeed, there are more natural choices for the equilibrium measure: the canonical and microcanonical Gibbs measures, for instance. We shall, however, prove the equivalence of the three descriptions in the context of infinite systems. On a thermodynamical level, the equivalence is well established in that Legendre transformations were shown to hold

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