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Intersections of Random Walks in Four Dimensions. II*

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Abstract. Let $f(n)$ be the probability that the paths of two simple random walks of length n starting at the origin in $\mathbb{Z}^4$ have no intersection. It has previously been shown that $f(n) \leqq c(\log n)^{-1/2}$. Here it is proved that for all $r > \frac{1}{2}$, $\lim_{n \to \infty} (\log n)^r f(n) = \infty$.

1. Introduction

Let $S_1(n, \omega)$ and $S_2(n, \omega)$ be independent simple random walks starting at the origin in $\mathbb{Z}^4$ (for definitions see [1]), and let Π_1, Π_2 denote the paths of the walks

$$\Pi_i(a, b) = \Pi_i(a, b, \omega) = \{S_i(n, \omega) : a < n < b\},$$
$$\Pi_i[a, b] = \Pi_i[a, b, \omega] = \{S_i(n, \omega) : a \leqq b \leqq b\},$$

and similarly for $\Pi_i(a, b]$ and $\Pi_i[a, b)$.

The probabilities that the paths Π_i intersect were studied in [1]. This paper follows up on that paper by giving a proof of a conjecture made. Let

$$f(n) = P\{\Pi_1[0, n] \cap \Pi_2(0, n] = \emptyset\}.$$

In [1] it was shown that there exist $c_1, c_2 > 0$ satisfying

$$c_1(\log n)^{-1} \leqq f(n) \leqq c_2(\log n)^{-1/2}, \tag{1.1}$$

and it was conjectured for $r > \frac{1}{2}$, that

$$\lim_{n \to \infty} (\log n)^r f(n) = \infty. \tag{1.2}$$

Here we prove (1.2).

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