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Supersymmetry and the Parisi-Sourlas Dimensional Reduction: A Rigorous Proof

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Abstract. Functional integrals that are formally related to the average correlation functions of a classical field theory in the presence of random external sources are given a rigorous meaning. Their dimensional reduction to the Schwinger functions of the corresponding quantum field theory in two fewer dimensions is proven. This is done by reexpressing those functional integrals as expectations of a supersymmetric field theory. The Parisi-Sourlas dimensional reduction of a supersymmetric field theory to a usual quantum field theory in two fewer dimensions is proven.

It was observed in the physics literature [1–5] that a D-dimensional classical field theory coupled to an external random source having Gaussian correlations is related order by order in perturbation theory to the corresponding $(D-2)$-dimensional quantum field theory without external sources.

This connection between random systems and the corresponding pure systems in two fewer dimensions was first noticed by Imry and Ma [1]. Investigating the effect of a quenched random magnetic field on phase transitions, they argued that classical mean field behavior occurred for dimensions greater than 6, instead of 4 as in non-random systems. Grinstein [2] found that the scaling laws for these random systems are the same as for pure systems, except that the dimension D is replaced by $D+2$.

Aharony, Imry, and Ma [3] explained those findings by arguing that the Feynman diagrams which give the leading singular behavior for the random case are identically equal, apart from combinatorial factors, to the corresponding Feynman diagrams for the pure case in two fewer dimensions. They showed this for one-loop diagrams that remain connected after all lines with random source insertions are opened. Young [4] extended their result for similar diagrams with an arbitrary number of loops.

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