Commun. Math. Phys. 85, 549–561 (1982)

Communications in Mathematical Physics
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Orbital Stability of Standing Waves for Some Nonlinear Schrödinger Equations

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Abstract. We present a general method which enables us to prove the orbital stability of some standing waves in nonlinear Schrödinger equations. For example, we treat the cases of nonlinear Schrödinger equations arising in laser beams, of time-dependent Hartree equations ....

Introduction

We consider here various nonlinear Schrödinger equations. To explain our results, we will give three examples.

1. Local Nonlinearities

$$i\frac{\partial\Phi}{\partial t}(t,x)+\Delta\Phi(t,x)+|\Phi(t,x)|^{p-1}\Phi(t,x)=0 \quad \text{in} \quad \mathbb{R}_+\times\mathbb{R}^N, \tag{1}$$

where $p>1$.

2. Hartree-Type Time Dependent Equations

$$\begin{aligned} i\frac{\partial\Phi}{\partial t}(t,x)+\Delta\Phi(t,x)+\sum_{i=1}^{m}\frac{z_i}{|x-x_i|}\Phi(t,x)-\left\{\int_{\mathbb{R}^3}|\Phi(t,y)|^2\frac{1}{|x-y|}dy\right\}\Phi(t,x)\\ =0 \quad \text{in} \quad \mathbb{R}_+\times\mathbb{R}^3, \end{aligned} \tag{2}$$

where $x_1,\ldots,x_m$ are some given points in $\mathbb{R}^3$ and $z_1,\ldots,z_m$ are positive constants.

3. Pekar-Choquard Time Dependent Equations

$$\begin{aligned} i\frac{\partial\Phi}{\partial t}(t,x)+\Delta\Phi(t,x)+\left\{\int_{\mathbb{R}^3}|\Phi(t,y)|^2\frac{1}{|x-y|}dy\right\}\Phi(t,x)\\ =0 \quad \text{in} \quad \mathbb{R}_+\times\mathbb{R}^3. \end{aligned} \tag{3}$$

0010-3616/82/0085/0549/$02.60