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Stability of Schrödinger Eigenvalue Problems

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Abstract. We derive a general stability criterion for discrete eigenvalues of Schrödinger operators, such as $A(\kappa)=p^2+V(x,\kappa)$, using only strong continuity of $A(\kappa)$ and $A^*(\kappa)$ in the perturbation parameter κ. The theory is developed for non-selfadjoint operators and illustrated with examples like the anharmonic oscillator, the Stark and the Zeeman effect. The principal tools are Weyl's criterion for the essential spectrum and a construction due to Enss [5]. They are also used to extend the classical invariance theorems for the essential spectrum to certain singular perturbations, including some local perturbations of the Laplacian by differential operators of arbitrary high order.

1. Introduction

We introduce the stability problems treated in this paper with the following familiar examples:

Example 1 (Anharmonic oscillator [11]).

$$A(\kappa)=p^2+x^2+\kappa x^4 \tag{1.1}$$

on $L^2(R^1)$, with complex κ in the sector

$$-\pi+\varepsilon\leqq\arg\kappa\leqq\pi-\varepsilon \qquad (\varepsilon>0). \tag{1.2}$$

Example 2 (Diatomic molecular ion [2]).

$$A(\kappa)=p^2-|x|^{-1}-|x-\kappa|^{-1} \tag{1.3}$$

on $L^2(R^3)$, with $\kappa\in R^3$.

Example 3 (Stark effect [6]).

$$A(\kappa)=e^{-2\theta}p^2-e^{-\theta}|x|^{-1}+\kappa e^{\theta}x_1 \tag{1.4}$$

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