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A Rigorous Block Spin Approach to Massless Lattice Theories*

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Abstract. The renormalization group technique is used to study rigorously the $\lambda(\nabla\phi)^4$ perturbation of the massless lattice field ϕ in dimensions $d \geqq 2$. Asymptoticity of the perturbation expansion in powers of λ is established for the free energy density. This is achieved by using Kadanoff's block spin transformation successively to integrate out high momentum degrees of freedom and by applying ideas previously used by Gallavotti and Balaban in the context of the ultraviolet problems. The method works for arbitrary semibounded polynomials in $\nabla\phi$ and $\Delta\phi$.

1. Introduction

During the last fifteen years renormalization group (RG) has become one of the main tools in both quantum field theory and statistical mechanics. Still, compared to the numerous works devoted to a variety of heuristic applications of this method, see e.g. the articles [1] and references therein, the rigorous results concerning RG are rather few, dealing with very simple models or attempting a rigorous formulation of the problems [2]. Here an exceptional position is occupied by the work of Gallavotti and collaborators [3, 4], where RG ideas were employed as a tool to prove ultraviolet stability of ϕ_3^4 quantum field theory. This allowed to turn RG into a powerful method in rigorous study of more complicated superrenormalizable field theory models, see [5] for an announcement of results concerning gauge theories.

In this paper we modify the technique developed by Gallavotti et al. and apply it to an infrared problem, namely the lattice model with hamiltonian density $\frac{1}{2}(\nabla\phi)^2 + \lambda(\nabla\phi)^4$, in dimensions $d \geqq 2$. The method works for arbitrary "irrelevant"

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