

The Resummation of One Particle Lines[★]

James Glimm¹ and Arthur Jaffe²

¹ The Rockefeller University, New York, New York 10021, USA

² Harvard University, Cambridge, Massachusetts 02138, USA

Abstract. We propose a partial resummation for a weak coupling cluster expansion. The resummation gives one particle lines with in/out field propagators. We give a Bethe-Salpeter equation in which one particle subtractions are defined using physical one particle states. By these methods, we show that $P(\phi)_2$ quantum fields in the weak coupling region have only isolated bound state spectrum below the $2m$ threshold. Here P is not restricted to be even.

1. Introduction

We present a new method for the study of mass spectrum, asymptotic completeness and related questions. The method uses exact subtractions of the physical one particle states; technically, it is based on the n -particle cluster expansion [8], and a (new)partial resummation which identifies the physical one particle lines in closed form.

Previous work [3, 4, 11, 12] on spectral properties of $P(\phi)_2$ quantum fields was based on the Euclidean program of defining n -particle irreducible amplitudes [13]. In this program, the vacuum subtractions are performed exactly, while one and higher particle subtractions are replaced by an orthogonalization of the polynomials in the Euclidean field ϕ . Thus in this program, a j -particle irreducible n -point function is defined by projecting $\phi(x_1)\phi(x_2)\dots\phi(x_n)$ onto the orthogonal complement, in the Euclidean Hilbert space $\mathcal{E} \equiv L_2(\mathcal{S}', d\mu)$, of the subspace spanned by the vectors

$$\phi(y_1)\dots\phi(y_l), l \leq j, y_v \in \mathbb{R}^2. \quad (1.1)$$

While the resulting subtraction agrees with the subtraction of (physical) j -particle intermediate states for $j=0$, it does not agree in general, and in particular if bound

[★] Supported in part by the National Science Foundation under Grants PHY 78-08066 and PHY 77-18762. Both authors thank the I.H.E.S., Bures-sur-Yvette, and A. J. thanks C.E.N., Saclay, for their hospitality.