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Axial Anomalies and Index Theorems on Open Spaces*

Constantine Callias

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Abstract: Using an approach inspired by the theory of the anomalous divergence of the axial vector current, we derive trace formulas for the resolvents of Dirac operators on open spaces of odd dimension. These formulas readily yield index theorems for these operators. As applications we determine the index of the Dirac operator for a particle of arbitrary isospin in the background field of a static system of SU(2) monopoles; and we find formulas in essentially closed form for certain determinants involving these operators.

Introduction

It is the purpose of this paper to give an elementary analytical proof of an index theorem for Dirac operators on open Euclidean spaces arising in Minkowski space-time Yang-Mills theories. We use basically an extension to open spaces of the index formulas in terms of traces of the propagation operators for the heat equation.

The index theorem has so far been applied to physical systems that can be stereographically projected onto a compact manifold [1]. Thus for the case of pseudoparticle configurations in Euclidean (imaginary time) Yang-Mills theory in four dimensions, it has been used to determine the number of zero eigenvalue modes of the Dirac equation and the number of independent parameters of self-dual or anti-dual solutions [2].

The possibility of compactifying the problem is the basic feature of these systems that makes the Atiyah-Singer [3] index theorem applicable. Compactification is actually necessary on a more fundamental level in these cases: to make the problem well-defined; precisely the operators involved are not Fredholm on the open space. The even dimensionality of the projected manifold guarantees that at least a priori the index problem can be non-trivial.

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