Commun. math. Phys. 59, 143—166 (1978)

Communications in
Mathematical
Physics

Some Probabilistic Techniques in Field Theory*

G. Benfatto, M. Cassandro, G. Gallavotti, F. Nicolò, E. Olivieri, E. Presutti, and E. Scacciatelli

Istituto di Matematica dell'Università dell'Aquila and
Istituto di Matematica e di Fisica dell'Università di Roma, I-00185 Roma, Italy

Abstract. We study, in the context of the Markov hyerarchical fields ($d=2$, 3) the role of the Markov property, of formal renormalization and of formal positivity. We determine upper and lower bounds for the ground state energy and discuss their relation with the perturbation theory series.

Introduction and Motivation

The basic property which allows to prove the rigorous validity of the perturbation expansion in euclidean field theory of φ^4 type in $d=2$, 3 space-time dimensions, is the "ultraviolet stability". The ultraviolet stability is the existence of a lower bound to the minimum of the spectrum of the renormalized Hamiltonian. In this paper we propose a model and a method of analysis which allows, in our opinion, to clarify the statistical mechanical aspects of the ultraviolet stability theorem. To motivate this model, and to illustrate the reasons which make it essentially as difficult as the euclidean field theory, we proceed as follows.

The euclidean field on R^d is a gaussian field with covariance

$$C=(1-D)^{-1} \tag{1}$$

where D is the Laplace operator on R^d. The ultraviolet divergences, originate from the divergence of the kernel $C_{\xi,\eta}$ of the operator C, as operator on $L_2(R^d)$, as $|\xi-\eta|\to 0$, if $d \geqq 2$. This remark leads to the idea [1], of representing C as

$$C=\sum_{N=0}^{\infty}\left[(2^{2N}-D)^{-1}-(2^{2(N+1)}-D)^{-1}\right] \tag{2}$$

and, correspondingly, the field $\tilde{\varphi}$ as,

$$\tilde{\varphi}_{\xi}=\sum_{N=0}^{\infty}\tilde{\varphi}_{\xi}^{[N]} \tag{3}$$

* This work has been partially supported by I.N.F.N., G.N.F.M., and G.N.S.M.