Commun. math. Phys. 57, 259—266 (1977)

Communications in
Mathematical
Physics

Correlation Inequalities for Ising Ferromagnets with Symmetries*

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Abstract. We derive new correlation inequalities for even Ising ferromagnets whose interaction is invariant under some symmetry transformation and satisfies a growth condition. The recent results of Schrader [1] and Messager and Miracle-Sol [2] for the nearest neighbour (n.n.) Ising model reappear as a special case. In addition we obtain monotonicity of $\langle\sigma_0\sigma_j\rangle$ under translation of j perpendicular to diagonal hyperplanes and the inequality $\langle\sigma_0\sigma_j\rangle \geqq \langle\sigma_0\sigma_{(\Sigma|j_\nu|,\underline{0})}\rangle$ for n.n. and other interactions.

1. Introduction

Recently, Schrader [1] and, independently, Messager and Miracle-Sol [2] found some new interesting inequalities for the n.n. Ising model. Let ϑ denote reflection,

$$\vartheta\sigma_j = \sigma_{(-j_1,\underline{j})},$$

and let $f_\nu(\sigma)$ be a polynomial in $\{\sigma_j; j_1 \geqq 0\}$, $\nu = 1, \ldots,$ with nonnegative coefficients. Then, for the n.n. Ising model [1]

$$\left\langle \prod_\nu (f_\nu \pm \vartheta f_\nu) \right\rangle \geqq 0 \tag{1.1}$$

for any combination of $\pm$ signs. In particular [1, 2], $\langle\sigma_0\sigma_{(j_1,\underline{j})}\rangle$ is monotone decreasing in $j_1 > 0$.

In this paper we analyze general even Ising ferromagnets which are invariant under some symmetry transformation, e.g., a reflection. Under a certain growth condition on the interaction we derive new correlation inequalities which contain those of [1, 2] as special cases (Theorems 3.1 and 3.2). The monotonicity properties of correlations which we obtain are of interest in themselves. We hope that the other inequalities may prove helpful to study the effects of boundary conditions as in [2] for the n.n. Ising model.

Let $\mathbb{Z}^d$ denote the d-dimensional square lattice with unit spacing. For $i \in \mathbb{Z}^d$, σ_i denotes the "spin at site i" with probability distribution given by some measure ν_i on

* Dedicated to Professor G. Ludwig's 60th birthday