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Are There Geon Analogues in Sourceless Gauge-Field Theories?*

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Abstract. It has recently been shown that there is no finite-energy non-singular solution to the sourceless gauge-field equations in four-dimensional Minkowski space that does not radiate energy. However, this does not preclude the possibility of solutions which hold themselves together for a long time before radiating away their energy. If they existed, such objects would be analogous to the geons of general relativity. We show such objects do not exist.

I. Introduction

This is the second of two papers devoted to the study of finite-energy non-singular solutions of classical gauge field theories in four-dimensional Minkowski space. In the first of these papers [1], it was shown that the only such solution that did not radiate energy to infinity was the vacuum solution, $F^a_{\mu\nu}=0$. Recently, this result has been strengthened by Weder [2], who showed that it is impossible for any non-zero amount of energy to be permanently confined in any compact volume.

These are strong results, but they do not exclude the possibility of energy being confined to some compact volume for a very long time before eventually escaping. A priori, this is a live possibility; after all, gauge field theories have much in common with general relativity, and general relativity is known to possess solutions of precisely this character, the geons of Wheeler [3], Brill and Hartle [4].

The purpose of this paper is to kill this possibility, and to attempt to understand physically why gauge field theories differ from general relativity in this respect.

In Section 2 we prove some rigorous theorems that can be (very loosely) described as saying that any configuration of gauge fields initially confined to the interior of some sphere falls apart in the time it takes light to cross the sphere. Not only are the forces of classical gauge field theory not enough to confine, they are not even enough to restrain.

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