Commun. math. Phys. 52, 175—189 (1977)

Communications in Mathematical Physics

On the ζ-Function of a One-dimensional Classical System of Hard-Rods

D. H. Mayer

Institut des Hautes Etudes Scientifiques, F-91440 Bures-sur-Yvette, France

K. S. Viswanathan

Department of Physics, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6

Abstract. The ζ-function of a one-dimensional classical hard-rod system with exponential pair interaction is defined as the generating function for the partition function of the system with periodic boundary conditions. It is shown, here, that the ζ-function for this system is simply related to the traces of the restrictions of the Ruelle's transfer matrix, and related operators to a suitable function space. This ζ-function does not, in general, extend to a meromorphic function.

Introduction

The new interest in classical one dimensional models of statistical mechanics has its origin in the work of Sinai [1] who found an interesting connection of these models with certain measure theoretic problems in the theory of dynamical systems. By constructing symbolic dynamics [2] for Anosov diffeomorphisms and flows on a compact manifold with the help of Markov partitions [3] he was able to apply the methods developed in the study of one dimensional models and to get interesting new results. A special role in the study of dynamical systems is played by the ζ-function of such a system introduced by Artin and Mazur [4]

$$\zeta(z)=\exp\left(\sum_{n=1}^{\infty} z^n N_n/n\right)$$

where N_n is the number of fixed points of the mapping f^n, where $f:M\to M$ is a diffeomorphism on some compact manifold M. They could show that the function $\zeta(z)$ has a non-vanishing radius of convergence for almost all diffeomorphisms f. To study the possible relevance of this ζ-function for statistical mechanics, Ruelle [5] introduced generalized ζ-functions in the following way:

Let M be some topological space and $f:M\to M$ a mapping. Let $A:M\to\mathbb{C}$ be a complex valued function on M. Then consider the formal expression

$$\zeta(z, e^A)= \exp\left[\sum_{n=1}^{\infty} \frac{z^n}{n}\left\{\sum_{x\in \mathrm{Fix}\, f^n}\left(\exp\sum_{k=0}^{n-1} A(f^k x)\right)\right\}\right]. \tag{1}$$