Commun. math. Phys. 44, 237—243 (1975)

Finite Mass Renormalizations in the Euclidean Yukawa$_2$ Field Theory*

Oliver A. McBryan

Department of Mathematics, Rockefeller University, New York, N. Y. 10021, USA

Received March 24, 1975

Abstract. We show that arbitrary finite boson mass renormalizations are possible in the Euclidean Yukawa$_2$ theory. We work in the Matthews-Salam representation with the fermions "integrated out".

I. Introduction and Results

We study the Yukawa$_2$ quantum field theory in a finite volume as a Euclidean boson field theory with the fermions "integrated out". The possibility of integrating out the fermions in the Yukawa theory was first demonstrated, in the external boson field case, by Matthews and Salam [1] and in the two-dimensional finite volume interacting theory, by Seiler [2] who showed that the resulting Fredholm determinants are L_p functions of the boson field. He thus obtained estimates on Schwinger functions of the form:

$$\begin{aligned} &|S(f_1, \dots, f_n; g_1, \dots, g_m; h_1, \dots, h_m)| \\ &\quad = |\langle \textstyle\prod_{i=1}^n \phi(f_i) \prod_{j=1}^m \Psi^{(1)}(g_j) \prod_{k=1}^m \Psi^{(2)}(h_k) e^{-V} \rangle| \\ &\quad \leqq c_1 c_2^{n+2m} (n!)^{\frac{1}{2}} \textstyle\prod_{i=1}^n \|f_i\|_{-1} \prod_{j=1}^m \|g_j\|_{-\frac{1}{2}} \|h_j\|_{-\frac{1}{2}}, \end{aligned} \tag{1.1}$$

with c_1, c_2 independent of ultraviolet cutoffs. The norms are those for the boson and fermion test-function spaces $\mathscr{H}_{-1}^{(\mu_0)}$, $\mathscr{H}_{-\frac{1}{2}}^{(m_0)} \otimes C^2$, where $\mathscr{H}_s^{(m)} = L_2(R^2, (k^2+m^2)^s d^2k)$.

In this paper we give a derivation of (1.1) even in the presence of an arbitrarily large negative boson mass renormalization. That such a renormalization is possible has already been demonstrated in the Hamiltonian formalism by Glimm [3, 4]. The basic idea is to decompose V into parts with high and low Fermi momenta. The high momentum part requires a "smaller" infinite ultraviolet mass renormalization than does the whole interaction, and the difference can then be used to dominate the finite boson mass renormalization. The low momentum part of e^{-V} may be expanded in a power series, since both fermions are bounded operators. Thus the Schwinger functions of the theory can be expressed as a power series in the Schwinger functions of the high momentum part. We prove a bound of the form (1.1) for the high momentum interaction and, applied to each term of the power series, this yields a bound of the same form for the full interaction. Our principal result is:

Theorem 1.1. *In the presence of a finite boson mass renormalization $-M$, a bound of the form* (1.1) *still applies, uniformly in the ultraviolet cutoffs, and the Schwinger functions converge as the cutoffs are removed.*

* Research supported in part by the National Science Foundation under Grant MPS 74-13252.