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On Local Functions of Fields*

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Abstract. Properties of local functions of fields are discussed. A condition, called the Borchers condition, is introduced which is weaker than duality but allows the construction of a maximal local extension of a system of local algebras. This extension will satisfy duality. The local structure of the generalized free field is studied, and it is shown that duality does not hold for the local algebras associated with certain generalized free fields, whereas the Borchers condition is satisfied for all generalized free fields. The appendix contains an elementary proof of duality for the free field.

1. Introduction

In the construction of local dynamical theories one usually considers Hamiltonian densities which are local functions of the basic fields. Equations of motion involve polynomials or other local functions of the fields. It is therefore of interest to investigate general properties which all local functions of fields should possess. The fields will be assumed to satisfy the restriction that the vacuum vector is analytic for the field operators, so that according to Borchers and Zimmerman [1] a system of local algebras $B_\phi(R)$ can be associated with each field $\phi(x)$. This means that an algebra $B_\phi(R)$ of bounded operators is associated with each region R and satisfies locality:

$$B_\phi(R_1) \subset B_\phi(R_2)' \quad \text{if} \quad R_2 \subset R_1{}^c . \tag{1}$$

Here $B_\phi(R)'$ denotes the commutant of $B_\phi(R)$ and R^c denotes the causal complement of R: the interior of the set of points space-like to all points in R. The local algebra $B_\phi(R)$ is constructed as the von Neumann algebra generated by the spectral projections of fields averaged with test functions with support in the region R.

In addition to the locality condition (1) a variety of restrictions on the local algebraic structure may be imposed. One such restriction of this sort is that of duality:

Definition 1. Duality: The duality condition holds for the region R if

$$B_\phi(R) = B_\phi(R^c)'$$

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