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Causally Continuous Spacetimes

S. W. Hawking

Institute of Astronomy, University of Cambridge, Cambridge, UK.

R. K. Sachs*

DAMTP, University of Cambridge, and Departments of Mathematics and Physics, University of California at Berkeley, Berkeley, USA

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Abstract. Causally continuous general relativistic spacetimes are defined and analyzed. In a causally continuous spacetime, the past and future of a local observer behave continuously under small perturbations of the metric or small changes in his location. Causally simple spacetimes are causally continuous; causally continuous spacetimes are causally stable. Possible reasons for taking causal continuity as a basic postulate in macrophysics are briefly discussed.

0. Introduction

Throughout this paper we consider macrophysics and use a non-quantum, general relativistic, time-oriented spacetime to model physical spacetime. What global properties does spacetime have? There seem to be three main possibilities.

First, there might conceivably be causality violations, even at the macroscopic level. Some important models, such as certain Kerr metrics, do have self-intersecting causal curves ([3, 9]). But macroscopic causality violations would imply a drastic alteration of standard physics. As yet, we have neither empirical evidence nor compelling theoretical arguments that causality violations occur.

Now suppose macroscopic causality violations cannot occur. We should presumably use models which remain causal even if the spacetime metric is perturbed slightly, since quantum and other limitations mean our macroscopic models and measurements are imprecise. Formally, this means using stably causal spacetimes ([7, 9]). Stable causality is perhaps the most plausible global assumption to make.

Finally, it may be that we should work exclusively with stably causal spacetimes which obey further restrictions. For example, globally hyberbolic spacetimes have a global Cauchy surface and are stably causal; asymptotically simple and empty spacetimes have, in addition, a behavior "at infinity" similar to that of Minkowski space ([11, 12, 9]). But observations, especially observations of the cosmological microwave background,

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