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The Inverse s-Wave Scattering Problem for a Class of Potentials Depending on Energy

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Abstract. The inverse scattering problem is considered for the radial s-wave Schrödinger equation with the energy-dependent potential $V^+(E,x) = U(x) + 2\sqrt{E}\,Q(x)$. (Note that this problem is closely related to the inverse problem for the radial s-wave Klein-Gordon equation of zero mass with a static potential.) Some authors have already studied it by extending the method given by Gel'fand and Levitan in the case $Q=0$. Here, a more direct approach generalizing the Marchenko method is used. First, the Jost solution $f^+(E,x)$ is shown to be generated by two functions $F^+(x)$ and $A^+(x,t)$. After introducing the potential $V^-(E,x) = U(x) - 2\sqrt{E}\,Q(x)$ and the corresponding functions $F^-(x)$ and $A^-(x,t)$, fundamental integral equations are derived connecting $F^+(x)$, $F^-(x)$, $A^+(x,t)$ and $A^-(x,t)$ with two functions $z^+(x)$ and $z^-(x)$; $z^+(x)$ and $z^-(x)$ are themselves easily connected with the binding energies E_n^+ and the scattering "matrix" $S^+(E)$, $E>0$ (the input data of the inverse problem). The inverse problem is then reduced to the solution of these fundamental integral equations. Some specific examples are given. Derivation of more elaborate results in the case of real potentials, and applications of this work to other inverse problems in physics will be the object of further studies.

1. Introduction

The problem of describing the interactions between colliding particles is of fundamental interest in physics. In many cases, a description can be carried out through a well known theoretical model. To this end, the following "inverse problem" is investigated: having determined some important quantities from experimental results, what are the values of the parameters occurring in the chosen model which reproduce them?

In particular, we are interested in collisions of two spinless particles, and we suppose that the s-wave scattering "matrix" $S(E)$ (defined for all energies $E>0$) and the s-wave binding energies E_n are exactly known from collision experiments.

We can try first to find a radial static potential $V(x)$ ($x \geqq 0$) which yields the given $S(E)$ ($E>0$) and E_n through the radial s-wave Schrödinger equation. We recall that this equation is written as follows, in the center-of-mass of the two particles, and with the usual reduced variables:

$$y'' + [E - V(x)]\,y = 0, \qquad x \geqq 0. \tag{1.1}$$

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