Commun. math. Phys. 3, 120—132 (1966)

On the Interaction Picture*

MARCEL GUENIN
Palmer Physical Laboratory, Princeton University, Princeton, N.J.

Received March 4, 1966

Abstract. It is shown that the difficulties connected with Haag's theorem can be bypassed, without losing Euclidean invariance, if the time evolution is considered as being only locally unitarily implementable.

A variant of the conventional interaction picture is defined, and a perturbation expansion derived which is shown to converge absolutely for a class of (non-trivial) Euclidean invariant 2-dimensional models.

1. Introduction

The Hamiltonian formalism, as applied to relativistic quantum field theories, has fallen very much into discredit during the last decade or so. But perhaps the time is now ripe to investigate again some of the basic difficulties of field theories, and this in the light of what has been learned in the so-called "axiomatic" approaches.

Following WIGHTMAN [1], we can label the three main difficulties of the conventional approach by the catchwords — Haag's theorem, — Instability of the vacuum, — Ultraviolet catastrophe. The aim of this paper is to show that the first of these difficulties, namely Haag's theorem, can be bypassed in a systematic way, with a slight alteration of the conventional formalism.

We shall have to define a new picture, closely related to the interaction (or Dirac) picture. The difference will be that the trivial part of the time evolution will be acting on the states (instead of on the operators) and the non trivial part will act on the operators. Haag's theorem says that the non-trivial part of the time evolution cannot be unitarily implemented (if the theory is Euclidian invariant), but we shall remark that it does not forbid us to have the time evolution acting as an algebraic mapping of a certain ring of operators. We shall even prove that this mapping can be locally unitarily implemented if it exists.

We consider the disagreeable feature of having the time evolution being only locally unitarily implementable (instead of globally) as very small in comparison to the advantage of being allowed to use the canonical commutation relations (instead of some inequivalent representation nobody knows how to construct) and to work in the Fock space.

* Research supported in part by the U.S. Air Force Office of Research, Air Research and Development Command.