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An Explicit Construction of the Quantum Group in Chiral WZW-Models

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Abstract: It is shown how a chiral Wess–Zumino–Witten theory with globally defined vertex operators and a one-to-one correspondence between fields and states can be constructed. The Hilbert space of this theory is the direct sum of tensor products of representations of the chiral algebra and finite dimensional internal parameter spaces. On this enlarged space there exists a natural action of Drinfeld's quasi-quantum group $A_{g,t}$, which commutes with the action of the chiral algebra and plays the rôle of an internal symmetry algebra. The R matrix describes the braiding of the chiral vertex operators and the coassociator Φ gives rise to a modification of the duality property.

For generic q the quasi-quantum group is isomorphic to the coassociative quantum group $U_q(g)$ and thus the duality property of the chiral theory can be restored. This construction has to be modified for the physically relevant case of integer level. The quantum group has to be replaced by the corresponding truncated quasi-quantum group, which is not coassociative because of the truncation. This exhibits the truncated quantum group as the internal symmetry algebra of the chiral WZW model, which therefore has only a modified duality property. The case of $g = su(2)$ is worked out in detail.

1. Introduction

A very important feature of two-dimensional conformal field theory is the fact that the theory "factorises" into a holomorphic and an anti-holomorphic theory. These two subtheories correspond essentially to the left- and right-movers of the original classical theory and are analytic (anti-analytic) in the sense that all correlation functions are meromorphic functions of the analytic (anti-analytic) parameters. Many properties of conformal field theory can be studied separately for the two chiral theories. This is of great importance as it allows the use of the powerful methods of complex analysis for the analysis of conformal field theory.

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