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Application of Dobrushin's Uniqueness Theorem to N-Vector Models

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Abstract. We apply Dobrushin's uniqueness theorem to N-Vector models to derive an upper bound of the critical temperature for unique equilibrium. In the case of isotropic ferromagnetic pair interactions this upper bound is the mean field critical temperature multiplied by a numerical factor.

1. Introduction

Recently, Driessler, Landau and Perez [1], with subsequent improvement by Simon [2], have established that N-Vector models with isotropic ferromagnetic pair interactions do not exhibit spontaneous magnetization for temperatures greater than the mean field critical temperature. In this paper we consider a related problem: to establish an upper bound for the critical temperature above which N-Vector models, with general interactions, exhibit a unique equilibrium state. Applying Dobrushin's uniqueness theorem [3] we derive an upper bound which, for isotropic ferromagnetic pair interactions, is the mean field critical temperature multiplied by a numerical factor of $\sqrt{5}$. For $N > 5$ this result improves a previous estimate of Simon [4] by a factor of $\sqrt{5/N}$.

2. Statement of Main Result

We consider the lattice model on $\mathbb{Z}^\nu$ with single spin space S^{N-1}, $N \geqq 2$. The configuration space of the lattice is the space of all functions $\sigma : \mathbb{Z}^\nu \to S^{N-1}$, denoted by $(S^{N-1})^{\mathbb{Z}^\nu}$. $(S^{N-1})^{\mathbb{Z}^\nu}$ is a topological space with product topology inherited from S^{N-1}. For $\sigma \in (S^{N-1})^{\mathbb{Z}^\nu}$, $\boldsymbol{\sigma}_a$ will denote the value of σ at lattice site a, and σ_a^i the i^{th} component of $\boldsymbol{\sigma}_a$ (with respect to the natural basis $\{\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \ldots, \hat{\mathbf{n}}_N\}$ of $\mathbb{R}^N$). The a priori measure μ_0 is the invariant probability measure on S^{N-1}.

To simplify the notation we presently consider only two-body and one-body

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