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Polynomial identities in ring theory, by Louis Halle Rowen, Academic Press, New York, 1980, xx + 365 pp., $39.50.

Rings satisfying a polynomial identity (PI) occupy an important place in modern noncommutative ring theory. Louis Rowen's book is an up to date, thorough presentation of PI-theory and related topics.

PI-theory was launched by a paper of Kaplansky in 1948 and "born again" with the discovery, by Formanek in 1971, of central polynomials. For convenience in this review we shall assume that our rings are algebras over fields. Thus, if A is an algebra over the field F, we say that A is a PI-ring if there exists a nonzero polynomial $p(x_1, \ldots, x_t)$ in noncommuting indeterminates with coefficients in F such that $p(a_1, \ldots, a_t) = 0$ for all possible substitutions of elements $a_1, \ldots, a_t$ in A. We say that p is an identity for A and that A is a PI-ring of degree d if d is the least degree of a polynomial which is an identity for A.

Commutative rings, obviously, satisfy $x_1x_2 - x_2x_1$. Nilpotent rings satisfy x^N where N is the index of nilpotence of the ring. Subalgebras and factor algebras of PI-rings are also PI-rings.

Before proceeding to the most important class of PI-rings, we pause for a definition. The nth standard polynomial S_n is $\Sigma_{\sigma \in \mathbb{S}_n} \operatorname{sgn}(\sigma) x_{\sigma(1)} \cdots x_{\sigma(n)}$ where the sum runs over the symmetric group, $\mathbb{S}_n$, on n letters and the x's are noncommuting variables. A celebrated theorem of Amitsur and Levitzki asserts that S_{2n} is an identity for the $n \times n$ matrices over a commutative ring. It is not difficult to see that $2n$ is the least degree of a polynomial identity for $n \times n$ matrices. Therefore, factor algebras of subalgebras of matrices are PI-rings–though not all PI-rings arise this way.

An important theme in the theory of PI-rings is the study of the "closeness" of classes of PI-rings to matrices over commutative rings and finding "tight" connections between PI-rings and their centers. Kaplansky proved that a primitive PI-ring is simple and finite dimensional over its center which is a field. Amitsur later showed that PI-rings with no nonzero nilpotent ideals are embeddable in matrices over commutative rings. To put this last result in perspective we note an observation of P. M. Cohn: the exterior algebra on an