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A SHORT PROOF OF THE DENJOY CONJECTURE

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1. In a recent letter Harold Shapiro has communicated to one of the authors his version of a nice (unpublished) proof of Matts Essén in which ideas from Heins' [2] version of the Denjoy-Carleman-Ahlfors theorem are applied to investigating the growth of certain univalent functions. It is the aim of the authors to show here that the direction may be reversed in a surprising way, giving a simple and natural proof of the Denjoy conjecture based on ideas from the theory of univalent functions. We wish to thank Harold Shapiro and Matts Essén for the inspiring communication.

Our proof of the D-C-A Theorem is based on a simple distortion theorem of a new type for univalent functions. This distortion theorem is interesting for its own right and has various extensions and applications, which we plan to publish at a later stage. Our distortion theorem may be proved either by path families methods or by inequalities for the logarithmic capacity. Although the former method gives sharper results and is also suitable for generalization to higher dimensions, we prefer the latter approach, because it is shorter and adequate for the proof of the

DENJOY-CARLEMAN-AHLFORS THEOREM. *If f is analytic in* $\mathbf{C}$ *and has* $n \geqslant 1$ *distinct and finite asymptotic values, then* $\liminf \log M(r) r^{-n/2} > 0$ *as* $r \longrightarrow \infty$.

2. Stars. By an *n-star,* $n \geqslant 1$, we mean a set S in $\mathbf{C}$, which is a union of n compact locally rectifiable Jordan arcs l_j, $1 \leqslant j \leqslant n$, called *arms,* with $l_j \cap l_k = \{0\}$ for $j \neq k$, such that $z = 0$ is the end point of all l_j. S is said to be *straight* if its arms are line segments, possibly of different lengths, but evenly spaced.

If g is a conformal map of $\hat{\mathbf{C}} \backslash S$ onto $\hat{\mathbf{C}} \backslash S'$ with $g(z) \longrightarrow 0$ as $z \longrightarrow 0$, we shall say shortly that g maps S conformally onto S'. If in addition $f(z) = z + \ldots$ near $z = \infty$ we shall say that f is normalized.

3. A distortion theorem. (i) *Every n-star S, $n \geqslant 1$, can be mapped onto a straight n-star S' by a normalized conformal map.*

(ii) *Let S be an n-star, $n \geqslant 1$, and $r > 0$, such that* $C(r) = \{z; |z| = r\}$

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