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RESEARCH ANNOUNCEMENTS

DEFINABLE DEGREES AND AUTOMORPHISMS OF $\mathcal{D}$

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The basic notion of relative computability, i.e. of one function $\alpha: N \longrightarrow N$ being computable from another β, defines, in the obvious way, first an equivalence relation $\alpha \equiv \beta$ on functions and then a partial ordering $\leqslant$, called Turing reducibility, on the equivalence classes, called Turing degrees. The analysis of the structure $\mathcal{D}$ of these degrees has been a central topic in recursion theory beginning with the papers of Post **[1944]** and Kleene and Post **[1954]**. We will here deal with a number of global or second order questions about $\mathcal{D}$ of the sort first raised in Rogers **[1967a]** and since then reiterated by many others. In particular we will show that many degrees and relations on them are definable purely in terms of the ordering answering some questions from Simpson **[1977]**. Our results also impose severe limitations on possible automorphisms of $\mathcal{D}$. Indeed every sufficiently large degree is fixed under every automorphism. (This answers questions from Rogers **[1967a]**, Simpson **[1977a]** and others.) By applying our methods to the principal filters (or cones) of $\mathcal{D}$ we can also considerably improve the solution to the homogeneity problem of Rogers **[1967]** given in Shore **[1979]** and **[1981]**. These results are all derived by combining a strengthening of Harrington and Kechris **[1975]** with the results and methods of Nerode and Shore **[1979]** and **[1980]** and Shore **[1979]**, **[1981]**. As in these latter papers a crucial role is played by the results of Jockusch and Soare **[1970]** on minimal covers and Lachlan **[1968]** on initial segments of $\mathcal{D}$.

LEMMA 1. *If the Turing degree* $\mathbf{x}$ *is not hyperarithmetic and* $\mathbf{x}, \mathcal{O} \leqslant t$ *then there is a degree* $\mathbf{s}$ *such that* $\mathbf{t}$ *is a minimal cover of* $\mathbf{s}$ *and* $\mathbf{x} \nleqslant \mathbf{s}$.

PROOF. Our starting point is the proof of Harrington and Kechris **[1975]** that every Π_1^0 set of reals A such that every hyperarithmetic real is recursive in some member of A contains reals of every Turing degree above $\mathcal{O}$. Let $A = \{\langle\sigma_0, \sigma_1, \tau_0, \tau_1\rangle | \sigma_1$ is a witness to the fact that every partial Π_1^1 function has a total extension recursive in σ_0 and τ_1 is the Skolem function witnessing that τ_0

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