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MARKOV PROCESSES AND RANDOM FIELDS[1]

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1. Introduction.

1.1. Suppose that a particle moves in a space E under the influence of random factors. Its position x_t at time t is a random variable, that is a measurable function on a space Ω where a probability measure P is given. The family $X = \{x_t\}$ is called a stochastic process in the state space E. It is important to evaluate the future behaviour of the particle using, in the best possible way, the information available at the present time. A stochastic process X is Markovian if, for a given value of x_{t_0}, the prognosis of the future does not depend on the evolution before t_0. A more symmetric form of the same property is: the families x_t, $t > t_0$ and x_t, $t < t_0$ are conditionally independent given x_{t_0}. During the past decades Markov processes became a powerful tool in partial differential equations and potential theory with important applications to physics.

Recently a growing interest is attracted by a generalization of stochastic processes known as random fields. A random field Φ over a space E is a family of random variables φ_x, $x \in E$. This is a mathematical model for systems with a large number of interacting random components which arise in physics, biology, sociology, theory of automata, etc.

A random field Φ over a space E has the Markov property on a pair of subsets B, C of E if the values of Φ on B and on C are conditionally independent given the values on the intersection $B \cap C$.

Investigation of the Markov property of a random field is closely related to the following prediction problem: To evaluate the values of the field on a set C by functionals of its values on a set B. A field has the Markov property on B, C if and only if the best estimate of values on C by values on B is a functional of values on $B \cap C$.

More precisely, we consider the Hilbert space $L^2(\Omega, P)$. Elements of this space which are determined by the values of Φ on B form a subspace $L(B)$. The best estimate of $Y \in L^2(\Omega, P)$ by an element of $L(B)$ is, geometrically, the orthogonal projection of Y on $L(B)$; in probabilistic language, it is called the conditional mathematical expectation of Y given Φ on B.

Suppose that random variables φ_x are real-valued and let H be the subspace of $L^2(\Omega, P)$ linearly generated by φ_x, $x \in E$. There exists an important class of fields, called Gaussian fields (see the definition at the

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