

IS COMPUTING WITH THE FINITE FOURIER TRANSFORM PURE OR APPLIED MATHEMATICS?

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HISTORICAL INTRODUCTION by L. Auslander

Let me begin with my view of a bit of history.

Before the Second World War mathematics in the United States was a servant of the needs of others and mathematicians taught service courses. Indeed, while A. Weil was teaching at an Eastern university it would be only a slight exaggeration to say that he was forbidden from presenting proofs in class and was called on the carpet by a dean for breaking this structure. In the years after the War, mathematics became a subject in its own right. Proofs became acceptable, as the creation of the "new math" proved to the world. Mathematicians were in demand, were men in their own right and no one's servants.

However, this growth period had a very unfortunate side affect. While mathematics was becoming a subject in its own right, many of its practitioners wanted to rid themselves of their former servant image. They had felt denigrated by the service role; so they denigrated service mathematics. Unfortunately, they lumped together service mathematics and applied mathematics. And so during this growth period of mathematics, there sprang up a distinction between pure and applied mathematics. During these years, the applied mathematicians felt the pure mathematicians looked down on them, and so the communications between the pure and applied mathematicians virtually dried up.

In this paper we will show that there is really not much difference between pure and applied mathematics. Indeed, we will cite instances of pure and applied mathematicians doing the same or analogous mathematics, but because of the lack of communication neither knew of the others' work.

With these broad generalities stated, let me try to explain how I came to the writing of this paper. This may perhaps serve as an example of how the gap between pure and applied mathematicians can be bridged.

I became interested in the study of the finite Fourier transform because I needed to know the eigenvalues of the finite Fourier transform. This arose in the study of the multiplicity of the regular representation of a solvmanifold. This problem was solved and the solution can be found in [8, p. 95]. Tolimieri, and Tolimieri and I, took up this problem in [18] and [3] and related the eigenvalue problem of the finite Fourier transform to a certain algebra of theta functions as discussed in Chapter I of this paper. I felt that

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