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SURJECTIVITY OF THE PERIOD MAP IN THE CASE OF QUARTIC SURFACES AND SEXTIC DOUBLE PLANES

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Let D be the period space of algebraic $K3$ surfaces, the generic point of which corresponds to either a sextic double plane or a quartic surface [1], [2].

THEOREM 1. *To every point of D, there corresponds a unique $K3$ surface.*

The uniqueness has been proved by Piatetskiĭ-Šapiro and Šafarevič. The proof of existence is outlined below.

Let o be the closed point of $A = \operatorname{Spec} \mathbf{C}[[t]]$. Let A_n be the finite covering of A obtained by extracting an nth root of t; let o_n be the closed point of A_n. *A family of surfaces over A_n* is a flat, projective map $f\colon X_n \longrightarrow A_n$ such that the generic fiber is smooth, connected, and two dimensional. The special fiber over o_n will be denoted by $\overline{X}_n$. A family of surfaces is said to have *ordinary singularities* if $\overline{X}_n$ is reduced and has nonsingular components crossing normally. A *modification* of a family $f\colon X \longrightarrow A$ is a family $f_n\colon X_n \longrightarrow A_n$ together with an isomorphism of the generic fiber of f_n with the pull-back of the generic fiber of f. Recall that a family of sextic double planes or quartic surfaces, $f\colon X \longrightarrow A$, induces a map of the generic point of A into D; the map extends to a map $\pi\colon A \longrightarrow D$ if and only if the monodromy group is finite [2]. The existence part of Theorem 1 follows from

THEOREM 1′.[1] *Given any family $f\colon X \longrightarrow A$ of sextic double planes such that the monodromy group is finite, there exists a modification $f_n\colon X_n \longrightarrow A_n$ for some n such that the special fiber $\overline{X}_n$ is reduced, irreducible and has at most rational double points as its singularities.*

THEOREM 1″. *Given any family $f\colon X \longrightarrow A$ of quartic surfaces such that the monodromy group is finite, there exists a modification $f_n\colon X_n \longrightarrow A_n$ for some n, with ordinary singularities, such that a component of $\overline{X}_n$ is a $K3$ surface whose periods correspond to the point $\pi(o)$ in D.*

It follows from Mumford's Geometric Invariant Theory [4] that in order

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[1] This theorem has also been proved by E. Horikawa [3].