BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 5, September 1976

# PHASE TRANSITIONS IN $P(\phi)_2$ QUANTUM FIELDS

BY J. GLIMM,[1] A. JAFFE[2] AND T. SPENCER[1]

Communicated by Daniel W. Stroock, May 17, 1976

Certain models in quantum field theory can be defined by a generalized random process $\phi(f) = \int \phi(x) f(x)\, dx$ for $f \in S(R^d)$ satisfying the following conditions [3]: (a) Regularity. The expectation of $e^{\phi(f)}$ is entire analytic on $S(R^d)$; (b) Euclidean invariance (including reflections) of the underlying measure $d\mu$. This means that

$$\int \left[\prod_i \phi(f_i)\right] d\mu = \int \left[\prod_i \phi(\eta f_i)\right] d\mu.$$

Here $(\eta f)(x) = f(\eta^{-1}x)$ and $\eta$ belongs to the Euclidean group. This identity induces a unitary transformation $T_\eta$ on the space $E = L_2(d\mu)$ of random variables. (c) Reflection positivity. Let $r$ denote reflection in the $x_0$ plane, and let $v$ be a function of the random variables $\{\phi(f)\}$ where suppt $f$ lies in the half space $x_0 > 0$. Then

$$\int \overline{v}(T_r v)\, d\mu \geqslant 0. \tag{2}$$

This final condition enables us to define the Hilbert space $H$ (which plays the role of $L_2$ of the state space) and a contraction semigroup $e^{-tH}$ (which defines the transition probabilities for the process). The inner product on $H$ is given by (2) after dividing out by the space of null vectors. The semigroup $e^{-tH}$ arises from translation in the $x_0 = t$ direction.

The simplest example of a process satisfying the above conditions is the Gaussian process whose generating functional is

$$\int e^{i\varphi(f)}\, d\mu_0 = \exp(-\tfrac{1}{2}\langle f, (-\Delta + 1)^{-1} f\rangle) L^2. \tag{3}$$

This process is known as the Ornstein-Uhlenbeck process. For $d = 1, 2$ we consider the following limiting process:

$$\int e^{i\phi(f)}\, d\mu^{\pm} = \lim_{\Lambda \uparrow R^2} \frac{\int e^{i\phi(f)} \exp(-P_\Lambda) \exp(-Q^{\pm}_{R^2\setminus\Lambda})\, d\mu_0}{\int \exp(-P_\Lambda)\exp(-Q^{\pm}_{R^2\setminus\Lambda})\, d\mu_0}. \tag{4}$$

---

*AMS (MOS) subject classifications* (1970). Primary 81A18.

[1]Supported in part by the National Science Foundation under Grant MPS74-13252.

[2]Supported in part by the National Science Foundation under Grant MPS75-21212.