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A WEINER-LIKE CONDITION FOR QUASILINEAR PARABOLIC EQUATIONS

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I consider the following parabolic equation

$$u_t = \operatorname{div} A(x, t, u, u_x) + B(x, t, u, u_x) \tag{1}$$

where A, B are respectively, vector and scalar valued measurable functions satisfying the structure conditions

$$\begin{gathered} |A(x, t, u, p)| \leqslant a_1 |p| + a_2 |u| + a_3, \quad |B(x, t, u, p)| \leqslant b_1 |p| + b_2^2 |u| + b_3^2, \\ p \cdot A(x, t, u, p) \geqslant c_1 |p|^2 - c_2^2 |u|^2 - c_3^2, \end{gathered} \tag{2}$$

where a_1, c_1 are positive constants, and all of the remaining coefficients a_i, b_i, c_i are in $L^{p,q}$ for some pair of numbers (p, q) satisfying $p \geqslant 2/(1-\theta)$; $n/p + 2/q \leqslant 1 - \theta$, where θ is a positive constant, $0 < \theta < 1$. This is precisely the equation studied by Aronson and Serrin [1] and is very similar to that studied by Trudinger [7].

We consider weak solutions from the class V^2 in cylinders $Q = \Omega \times (0, T)$ where $\Omega \subset \mathbf{R}^n$ is a bounded domain. $V^2(Q)$ is defined to be the space of measurable functions u which have finite norm

$$\|u\|_{V^2(Q)} = \operatorname*{ess\,sup}_{0<t<T} \left\{ \int_\Omega |u(x, t)|^2 \, dx \right\}^{1/2} + \sum_{i=1}^n \left\| \frac{\partial u}{\partial x_i} \right\|_{L^2(Q)}$$

where $\{\partial u/\partial x_i\}_{i=1,\dots,n}$ are the weak (i.e. distributional) derivatives of u. We define $V_0^2(Q)$ to be the colsure in $\| \cdot \|_{V^2(Q)}$, of functions in $C^\infty(Q)$ which vanish in a neighborhood of the parabolic boundary $\partial_p Q = \overline{\Omega} \cup \{\partial\Omega \times [0, T]\}$. We say that $u \in V^2(Q)$ is a weak solution to (1) if $\int \varphi_t u - \varphi_x \cdot A(x, t, u, u_x) + \varphi B(x, t, u, u_x) = 0$ for every function $\varphi \in C_c^\infty(Q)$.

The Maximum Principle for such equations (Aronson and Serrin [1, Theorem 1], can be generalized to the notion of weak boundary values as follows.

THEOREM . *If $u \in V_0^2(Q)$ is a weak solution to* (1) *then almost everywhere in Q we have*

$$|u(x, t)| \leqslant C(\|b_3\|_{p,q}^2 + \|c_3\|_{p,q})$$

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