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COMPLETION AND EMBEDDING BETWEEN PSEUDO (v, k, λ)-DESIGNS AND (v, k, λ)-DESIGNS

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ABSTRACT. Each of four arithmetical conditions on the parameters v, k, and λ of a given primary pseudo (v, k, λ)-design is necessary and sufficient to ensure completion or embedding between the given design and some (v', k', λ')-design.

Let $X=\{x_1, \cdots, x_v\}$, and let $X_1, \cdots, X_v$ be subsets of X. The subsets $X_1, \cdots, X_v$ are said to form a (v, k, λ)-*design* if

each X_j $(1\leqq j\leqq v)$ has k elements;
any two distinct X_i, X_j $(1\leqq i, j\leqq v)$ intersect in λ elements; and
$0\leqq\lambda<k<v-1$.

Such a design is completely determined by its *incidence matrix*; this is the $(0, 1)$-matrix $A=[a_{ij}]$ defined by taking $a_{ij}=1$ if $x_j \in X_i$ and $a_{ij}=0$ if $x_j \notin X_i$. More information about these combinatorial designs is available, for example, in [2] and [5].

Let $Y=\{y_1, \cdots, y_v\}$, and let $Y_1, \cdots, Y_{v-1}$ be subsets of Y. The subsets $Y_1, \cdots, Y_{v-1}$ are said to form a *pseudo* (v, k, λ)-*design* if

each Y_j $(1\leqq j\leqq v-1)$ has k elements;
any two distinct Y_i, Y_j $(1\leqq i, j\leqq v-1)$ intersect in λ elements; and
$0<\lambda<k<v-1$.

The incidence matrix of a pseudo (v, k, λ)-design is defined in the same manner as the incidence matrix of a (v, k, λ)-design.

The consideration of pseudo (v, k, λ)-designs was suggested during the course of study of "modular hadamard matrices" [3], [4]. Related work has been published by Bridges [1] and Woodall [6].

A pseudo (v, k, λ)-design is "almost" (its incidence matrix lacks one row) a (v, k, λ)-design; this suggests the consideration of "completion and embedding" between these two combinatorial designs. Let A be the incidence matrix of a pseudo (v, k, λ)-design. Then it *might* be possible to

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