BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 6, November 1973

PIECEWISE POLYNOMIALS AND THE FINITE ELEMENT METHOD[1]

BY GILBERT STRANG

Thirty years ago, Courant gave a remarkable lecture to this Society. My talk today is more or less a progress report on an idea which he described near the end of that lecture. There are a lot of people in this city, and a few in this room, who worked very closely with Courant—but the idea I am talking about came to fruition in a different and more unexpected way.

To begin with, his idea was forgotten. Perhaps you have forgotten it too; it had to do with approximation by piecewise polynomials, and I will try to explain it properly in a moment. Ten years later Pólya made a very similar suggestion [3], [4], without reference to Courant's lecture. At the same time, and independently, Synge did exactly the same thing [**10**]. Meanwhile Schoenberg had written the paper [5] which gave birth to the theory of splines—again proposing that, for approximation and interpolation, the most convenient functions were piecewise polynomials.

Certainly there was an idea whose time was coming. When it finally came, fifteen years after Courant's lecture, it developed into what is now the most powerful technique for solving a large class of partial differential equations—the *finite element method*. The only sad part is that virtually the whole development took place as if Courant had never existed. It is like the story of Romulus and Remus (I think); in this case, the wolves who eventually took care of the orphan happened to be structural engineers.[2] They needed a much better technique for the solution of complicated elliptic systems, and in numerical analysis the algorithms which survive and mature are those which are needed. We want to describe this finite element method, and then at the end to propose an open problem; its interest may be more algebraic-combinatorial than practical, but it is directly suggested by the construction of finite elements.

Prior to Courant, the usual approximating functions were sines and cosines, or Bessel functions, or Legendre polynomials. For a simple problem on a regular domain, these are still completely adequate; their approximation properties are well known, and integrations are simplified

---

AMS(MOS) subject classifications (1970). Primary 35J20, 41A15, 41A25, 65N30.

Key words and phrases. Partial differential equations, finite element method, approximation, piecewise polynomials.

[1] The preparation of this lecture, delivered to the American Mathematical Society in New York City on April 20, 1973, was supported by the National Science Foundation (P22928).

[2] I hope they will forgive the analogy.