BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 6, November 1972

ON CHARACTERISTIC CLASSES OF Γ-FOLIATIONS

BY R. BOTT[1] AND A. HAEFLIGER

Communicated by Michael Altiyah, April 13, 1972

1. **Introduction.** In this note we describe and apply a general procedure for constructing characteristic classes of Γ-foliations. Our work was inspired by that of Godbillon-Vey [4] and Gelfand-Fuks [5] on the one hand, and that of Chern-Simons [3] on the other.

We know of several essentially equivalent procedures—some more in the line of [3] (see for example [2])—but the one we present here fits naturally and well nigh unavoidably into the Gelfand-Fuks theory and we have been told that Malgrange and Gelfand have also, quite independently, come upon it.

We will take the following point of view toward characteristic classes and foliations. Consider a pseudogroup Γ whose elements are diffeomorphisms of open sets in $\boldsymbol{R}^n$. A Γ-foliation on a smooth (that is C^∞) manifold M is by definition a maximal family F of submersions

$$f_U : U \to \boldsymbol{R}^n$$

of open sets U in M, such that for every $x \in U \cap V$ there exists an element $\gamma_{VU} \in \Gamma$ with $f_V = \gamma_{VU} \circ f_U$ in some vicinity of x.

Given two Γ-foliations, F on M and F' on M', a morphism from F to F' is by definition a smooth map

$$f : M \to M'$$

such that for every "local projection" $f'_U \in F'$ the composition

$$f'_U \circ f : f^{-1}U \to \boldsymbol{R}^n$$

is in F.

With this concept of morphism the Γ-foliations form a category $C(\Gamma)$ and we define a characteristic class of Γ-foliations with coefficients in a group $\boldsymbol{R}$, as a natural transformation

$$\alpha : C(\Gamma) \to H^*(\ ;\boldsymbol{R}).$$

Thus $\alpha(F) \in H^*(M;\boldsymbol{R})$ and if $f : F' \to F$ is a morphism, then

AMS 1970 *subject classifications*. Primary 53A10, 53C35; Secondary 53A05, 28A75.

[1] Supported in part by National Science Foundation grant GP-7952X3, while at the Institute for Advanced Study, Princeton, New Jersey.