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DOUBLE CENTRALIZERS OF PEDERSEN'S IDEAL OF A C^*-ALGEBRA

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1. **Introduction.** The theory of double centralizers was developed for topological algebras by Johnson in [9] and [10] and further investigated in the case of a C^*-algebra by Busby [2]. If A is a commutative C^*-algebra, that is $A = C_0(X)$—the algebra of all complex valued continuous functions which vanish at infinity on a locally compact Hausdorff space X—then the algebra of all double centralizers of A is $C_b(X)$—all the bounded compley continuous functions on X [10]. The noncommutative generalization of the relationship between $C_0(X)$ and $C_b(X)$ was found useful by Busby in his papers on extensions of C^*-algebras ([2], [3]). The purpose of this note is to construct and to investigate a noncommutative analogue of $C(X)$, the algebra of all complex continuous functions on X. Let $C_c(X)$ be the ideal of $C_0(X)$ consisting of all the functions with compact support. Then the algebra of all double centralizers of $C_c(X)$ can be identified with $C(X)$ [10]. Thus, a way to find a generalization of the algebra $C(X)$ is by using an ideal of a C^*-algebra which plays a similar role to that of $C_c(X)$ in $C_0(X)$. Such an ideal was shown to exist in any C^*-algebra by Pedersen [12] and we shall exploit its properties towards the above stated aim. The full details of our discussion will appear elsewhere and we intend to pursue the matter in subsequent papers.

We refer the reader to the papers [2], [9] and [11] for the definitions and the main facts concerning double centralizers and Pedersen's ideal. From now on A will denote a C^*-algebra. We shall denote its Pedersen's ideal by K_A or simply by K if the C^*-algebra under consideration is well understood. The algebra of all double centralizers of A (respectively, K) is denoted by $\Gamma(A)$ (respectively, $\Gamma(K)$). The subalgebra of $\Gamma(K)$ consisting of all double centralizers (S, T) for which S, T are bounded will be denoted by $M(K)$. When convenient, we shall identify A, respectively K, with their canonical images in $\Gamma(A)$, respectively $M(K)$ [9]. If $B \subset A$ then $B^+ = \{a \in B : 0 \leqq a\}$.

2. In this section we shall present a few simple properties of K and $\Gamma(K)$ which are useful in many of our proofs.

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