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BOUNDARY VALUE PROBLEMS FOR QUASILINEAR ELLIPTIC EQUATIONS WITH RAPIDLY INCREASING COEFFICIENTS

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1. **Introduction.** The purpose of this note is to present a general existence theorem for variational boundary value problems for quasilinear elliptic operators in divergence form:

$$A(u) = \sum_{|\alpha| \leqq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, \ldots, \nabla^m u), \tag{1}$$

in the case where the coefficients A_α do not have polynomial growth in u and its derivatives. The crucial points in the treatment of rapidly (or slowly) increasing A_α's are that the Banach spaces in which the problems are appropriately formulated are nonreflexive and that the corresponding operators are not bounded nor everywhere defined and do not generally satisfy a global a priori bound. This existence theorem is based upon an extension of the theory of not everywhere defined unbounded pseudo-monotone mappings (Browder [5], [6], Browder-Hess [7]) to the context of complementary systems.

Detailed proofs will appear elsewhere.

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2. **Main results.** We will use the following notations. If $\xi = \{\xi_\alpha : |\alpha| \leqq m\} \in R^{s_m}$ is a m-jet, then $\zeta = \{\xi_\alpha : |\alpha| = m\} \in R^{s'_m}$ denotes its top order part and $\eta = \{\xi_\alpha : |\alpha| < m\} \in R^{s_{m-1}}$ its lower order part; for u a derivable function, $\xi(u)$ denotes $\{D^\alpha u : |\alpha| \leqq m\}$. The Orlicz space **[11]** on $\Omega \subset R^n$ corresponding to an N-function M is denoted by $L_M(\Omega)$ and the closure in $L_M(\Omega)$ of the simple functions in Ω by $E_M(\Omega)$. The Sobolev space of functions u such that u and its distribution derivatives up to order m lie in $L_M(\Omega)$ $[E_M(\Omega)]$ is denoted by $W^m L_M(\Omega)$ $[W^m E_M(\Omega)]$; these spaces are identified to subspaces of the product $\prod_{|\alpha| \leqq m} L_M(\Omega) \equiv \prod L_M$. $\overline{M}$ $[M^{-1}]$ denotes the function conjugate [reciprocal] to M and $N \ll M$ means that, for each $\varepsilon > 0$, $M(\varepsilon t)/N(t) \to +\infty$ as $t \to +\infty$.

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