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ON HERMITIAN STRUCTURES OF PRESCRIBED NONPOSITIVE HERMITIAN SCALAR CURVATURE

BY MELVYN S. BERGER

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1. **Introduction.** Let (M, g) be a given Kähler metric on a compact manifold M of complex dimension N. If one denotes the associated volume element by dV_g and the scalar curvature by k_g, then it is known [**1**, p. 118] that $c(M) = \int_M k_g\, dV_g$ is a Kähler invariant (i.e. is independent under Kähler deformation of the particular Kähler metric g defined on M). Here we give some results on the following problem:

(π_N) Find necessary and sufficient conditions on a Hölder continuous function $K(x)$ defined on (M, g) such that $K(x)$ is the Hermitian scalar curvature of some Hermitian metric $\bar{g}$ on M conformally equivalent to $\bar{g}$.

If $N = 1$, this problem was studied by the author in [**2**], and subsequently in [**3**] by Kazdan and Warner. However the methods used in these papers depend crucially on the fact that $N = 1$. Indeed, certain calculus inequalities for the functions u in the Sobolev space $W_{1,2}(M, g)$ are required, that hold in case $N = 1$ but not otherwise.

2. **The main result.** We seek a smooth real-valued function $u(x)$ defined on (M, g) such that the Hermitian scalar curvature $k_{\tilde{g}}(x)$ of M with respect to the Hermitian metric $\tilde{g} = e^{2u}g$ is the given function $K(x)$. By means of the results of Chern [**4**], and Chavel [**5**], the function $u(x)$ is a solution of the semilinear elliptic equation

$$N\Delta u - k_g(x) + K(x)e^{2u} = 0$$

where Δ is the Laplace-Beltrami operator relative to (M, g). By integrating (1) over M, we find that a necessary condition for the solvability of (1) is that

$$c(M) \equiv \int k_g(x)\,dV_g = \int K(x)e^{2u}\,dV_g.$$

By the remarks of the Introduction, this relation is invariant under Kähler deformation of g, and is an analogue of the Gauss-Bonnet formula for $N = 1$. As in [**2**], (2) may be used to formulate isoperimetric variational problems whose solutions (if they exist) satisfy (1). However, if $N > 1$, the solvability of these isoperimetric problems is in question, so an

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