BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

RESEARCH ANNOUNCEMENTS

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OPERATORS ON FUNCTION SPACES

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Communicated by Robert Bartle, March 10, 1972

1. **Introduction.** In this announcement we present characterizations of weakly compact and compact operators defined on function spaces. Besides the space of totally measurable functions, we consider the space of all Banach-valued continuous functions, where the topology of the space is either the compact-open topology or the topology generated by the supremum norm on functions vanishing at infinity. The main tools are a recent result of Brooks [5] concerning weak compactness of vector measures, and integral representation theorems in a very general setting [8] which serve to unify the existing theorems of this type and facilitate the study of operator theory. Our characterization provides a natural and simple condition for operators to be weakly compact—namely that $\tilde{m}(A_i) \to 0$, whenever $A_i \searrow \emptyset$, where $\tilde{m}$ is the semivariation of the representing measure for the operator. This extends the Bartle-Dunford-Schwartz theory [2] for weakly compact operators from $C(S)$ into X. The necessity part of Theorem 1 extends the work of Batt and Berg [4]. Also we give a necessary and sufficient condition, in terms of the underlying topology of the domain space, in order that the classes of weakly compact and compact operators coincide. Finally in §4 we briefly mention additional results concerning operators. In a later paper [7], representations in the setting of locally convex spaces and applications will be given.

AMS 1969 *subject classifications*. Primary 4625, 4725; Secondary 2850.
Key words and phrases. Compact operator, weakly compact operator, representation theorem, strongly bounded vector measure, dispersed space.
[1] The first author was supported in part by NSF grant GP-28617.
[2] The second author was supported in part by Faculty Research Grant 34530, North Texas State University.