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AN EXISTENCE THEOREM FOR A MODIFIED SPACE-INHOMOGENEOUS, NONLINEAR BOLTZMANN EQUATION

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1. **Preliminaries.** Consider the initial value problem for the following class of nonlinear Boltzmann equations:

$$\frac{\partial}{\partial t} f + v\nabla_x f = Qf \qquad (t > 0), f(0) = f_0 \geqq 0, \tag{1.1}$$

with collision operator

$$Qf(x, v_1) = \int_{R^3 \times B^2} [J_u^* \langle f_1 f_2 \rangle - \langle f_1 f_2 \rangle] k \, du \, dv_2. \tag{1.2}$$

The space coordinate x belongs to a parallelepiped

$$\omega = \{x = (x_1, x_2, x_3); |x_j| \leqq a_j/2\},$$

and

$$\begin{aligned} \langle f_1 f_2 \rangle &= f(x, v_1) f(x, v_2) && \text{if } |f(x, v_1) f(x, v_2)| \leqq N, \\ &= N \operatorname{sign} f(x, v_1) f(x, v_2) && \text{otherwise.} \end{aligned}$$

The impact parameter u in R^2 is, for convenience, restricted to the disc

$$B^2 = \{u \in R^2; |u| \leqq 1/\sqrt{\pi}\}.$$

The kernel $k(v_1, v_2)$ is a measurable, nonnegative, and bounded function vanishing for $|v_1|^2 + |v_2|^2 > K_1^2$, with $k(v_1, v_2) = k(v_2, v_1)$ and invariant under $J^*, J^*k = k$. Here J^* is induced by a C^1-diffeomorphism J on $R^3 \times R^3 \times B^2$, restricted in a certain way. With the velocity mappings

$$\begin{aligned} &1\colon\ R^3 \times R^3 \to R\colon (v_1, v_2) \to 1, \\ &p\colon\ R^3 \times R^3 \to R^3\colon (v_1, v_2) \to v_1 + v_2, \\ &T\colon\ R^3 \times R^3 \to R\colon (v_1, v_2) \to |v_1|^2 + |v_2|^2, \\ &\Sigma\colon\ R^3 \times R^3 \to R^3 \times R^3\colon (v_1, v_2) \to (v_2, v_1), \end{aligned}$$

the restriction on the (collision) mapping J can be written

$$1 \circ J_u = 1, \qquad p \circ J_u = p, \qquad T \circ J_u = T, \tag{1.3}$$

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