BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 2, March 1972

A TERMINAL COMPARISON PRINCIPLE FOR DIFFERENTIAL INEQUALITIES[1]

BY THOMAS G. HALLAM

Communicated by Fred Brauer, October 8, 1971

1. **Introduction.** The comparison principle has proved to be very useful in the study of various qualitative problems in ordinary differential equations. Comparison principles have been previously formulated in terms of initial value problems and, in this setting, their applications are numerous [**6**]. In this announcement, a comparison principle for terminal value problems is given. Related topics and applications are also discussed.

A comparison principle for terminal value problems has been stated in [7]; however, the proof given there for the weak inequality case needs some modification. In fact, the validity of a terminal comparison principle in the full generality of the statement in [7] remains an open question.

2. **Preliminary hypotheses and definitions.** A solution of the initial value problem which consists of the differential equation

$$dr/dt = g(t, r) \tag{1}$$

and the point (t_0, r_0) will be denoted by $r(t, t_0, r_0)$. In (1), it is assumed that $g \in C[R_+ \times R, R]$. It will be tacitly assumed that given any $r_0 \in R$, there exists a $t_0 \in R_+$ so that the solution $r(t, t_0, r_0)$ of (1) exists on $[t_0, \infty)$ and $\lim_{t\to\infty} r(t, t_0, r_0)$ exists. A solution of the terminal value problem consisting of equation (1) and a terminal value r_∞ will be denoted by $r(t; r_\infty)$.

A solution $r_m(t; r_\infty)$ is a (the) *maximal solution of the terminal value problem* $\{(1); r_\infty\}$ *on the interval* I provided any other solution $u(t; r_\infty)$ of the terminal value problem $\{(1); r_\infty\}$ which is valid on I satisfies the inequality

$$u(t; r_\infty) \leqq r_m(t; r_\infty) \qquad (t \in I).$$

A similar definition may be given for the *minimal solution* of a terminal value problem. Either of the above types of solutions will be referred to as an *extremal solution* of the terminal value problem.

For initial value problems, the hypothesis $g \in C[R_+ \times R, R]$ is sufficient to insure the existence of maximal and minimal solutions [**6**, p. 11]. However, even if the continuity of g in t is extended to the interval $[0, \infty]$,

---

AMS 1970 *subject classifications*. Primary 34A40, 34C10.
Key words and phrases. Comparison principle, differential inequalities, maximal and minimal solution, terminal value problem, asymptotic equilibrium.
[1] This research was supported in part by the National Science Foundation under Grant GP-11543.