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QUOTIENTS OF FINITE W^*-ALGEBRAS[1]

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1. In this note we present results concerning the following problem. Suppose M is a W^*-algebra and $J \subset M$ a uniformly closed two-sided ideal. Then the quotient algebra M/J is a C^*-algebra, and the problem is: What are the conditions that M/J be a W^*-algebra?

Since we can write M as a direct sum of a finite and a properly infinite W^*-algebra, we can discuss the two cases separately. In [3] and [4] Takemoto solved the problem for a properly infinite W^*-algebra, that can be represented on a separable space. His theorem states that M/J is a W^*-algebra, if and only if J is ultra-weakly closed.

2. If M is finite the situation is quite different. There are "many" non-ultra-weakly closed ideals J for which the quotient M/J is a W^*-algebra. Indeed, Wright [5] and Feldman [1] proved that if J is a maximal ideal, M/J is a finite factor. This result was proved by a different method by Sakai in [2]. The following theorem generalizes that result.

THEOREM 1. *Let M be a finite and σ-finite W^*-algebra with center Z. Let J be a uniformly closed two-sided ideal satisfying the following conditions:*

(i) *J is an intersection of maximal ideals,*

(ii) *$Z/Z\cap J$ is a W^*-algebra,*

(iii) *$Z/Z\cap J$ is σ-finite.*

Then M/J is a W^*-algebra.

As a partial converse we have

THEOREM 2. *If J is a uniformly closed two-sided ideal of the finite and σ-finite W^*-algebra M and M/J is a W^*-algebra, then J satisfies the conditions* (i) *and* (ii) *of Theorem* 1.

REMARK. If we assume that M can be represented on a separable

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[1] The results are contained in the author's doctoral dissertation at the University of Pennsylvania, and his supervisor is S. Sakai. The author is a research fellow at the University of Aarhus, Denmark.