REPRESENTING A MEROMORPHIC FUNCTION AS THE QUOTIENT OF TWO ENTIRE FUNCTIONS OF SMALL CHARACTERISTIC

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We announce the following result and sketch the method of proof.

THEOREM. *There exist absolute constants A and B such that if f is any meromorphic function in the complex plane, then there exist entire functions g and h such that $f=g/h$ and such that $T(r, g) \leqq AT(Br, f)$ and $T(r, h) \leqq AT(Br, f)$ for all $r>0$.*

Here $T(r, f)$ is the Nevanlinna characteristic of f evaluated at r.

Rubel and Taylor [1] have obtained such a representation for certain special classes of meromorphic functions. It is shown in [1] that both the arguments and the moduli of the zeros and poles of f play an important role in such a representation; the proof of the above theorem is based on results in [1] together with a new technique for "balancing" the zeros and poles of f.

We now sketch the proof. Let $Z=\{z_n\}$ be the set of poles of f listed according to multiplicity. Without loss of generality we may assume $f(0)\neq\infty$. Let

$$n(t, Z) = \sum_{|z_n|\leq t} 1 \quad \text{and} \quad N(r, Z) = \int_0^r \frac{n(t, Z)}{t}\,dt$$

From [1] it is sufficient to show that there exist absolute constants A' and B' and a set $\tilde{Z}=\{\tilde{z}_n\}$ containing Z such that for all $r>0$,

$$N(r, \tilde{Z}) \leqq A'T(B'r, f) \tag{1}$$

and such that for all positive integers k and all $s>r\geqq 1$,

$$\left|\frac{1}{k}\sum_{r<|\tilde{z}_n|\leq s}\left(\frac{1}{\tilde{z}_n}\right)^k\right| \leqq \frac{A'T(B'r, f)}{r^k} + \frac{A'T(B's, f)}{s^k}. \tag{2}$$

$\tilde{Z}$ is constructed in the following way. For each integer $N\geqq 1$, we consider those $z_n\in Z$ such that $2^N<|z_n|\leqq 2^{N+1}$ and relabel them simply $z_1, z_2, \cdots, z_{p_N}$ with $z_j=|z_j|e^{i\theta_j}$ and $|z_j|=2^{N+\alpha_j}$ where $0<\alpha_j\leqq 1$ for $1\leqq j\leqq p_N$. For notational convenience we do not indicate the obvious dependence of z_j, θ_j, and α_j on N. We define

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