EXTENSION THEORY FOR CONNECTED HOPF ALGEBRAS

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1. **Introduction.** Let K be a fixed commutative ring with unit. We will deal with graded algebras, coalgebras, and Hopf algebras over K as defined in Milnor-Moore [4], but we assume that the underlying K-modules are connected.

Suppose A, B are Hopf algebras; A commutative and B cocommutative. By an extension of B by A we mean a diagram of Hopf algebras and Hopf maps

$$A \xrightarrow{\alpha} C \xrightarrow{\beta} B \tag{1.1}$$

in which C is isomorphic to $A \otimes B$ simultaneously as a left A-module and right B-comodule. In this paper we announce results which describe and classify all extensions by B by A. Proofs will appear in [5].

2. **Matched pairs.** If B is an algebra we write

$$\eta : K \to B, \qquad \mu_B : B \otimes B \to B$$

for the unit and multiplication, respectively. If A is a coalgebra we write

$$\epsilon : A \to K, \qquad \psi_A : A \to A \otimes A$$

for the counit and comultiplication.

As the first step in classifying extensions, we will show in [5] how a diagram (1.1) determines a pair of K-linear maps

$$\sigma_A : B \otimes A \to A, \qquad \rho_B : B \to A.$$

σ_A is the "action" of base on fiber that one expects in an extension problem; ρ_B is its dual. We prove:

(a) σ_A gives A the structure of a left B-module algebra;

(b) ρ_B gives B the structure of a right A-comodule coalgebra;

(c) the diagram commutes:

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