A NOTE ON THE ROYDEN BOUNDARY

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Recently Loeb and Walsh [3] established several results concerning the Royden boundary in the axiomatic setting. This in effect generalizes theorems about HD-functions, Dirichlet-finite harmonic functions on surfaces to Riemannian manifolds, which are the most general carriers of these functions. Their results are however restricted to bounded HD-functions. Whether Nakai's [4] characterization of the HD-functions in terms of the Royden boundary is valid for Riemannian manifolds remains open. The purpose of this note is to announce that Nakai's theory does carry over in its entirety.

The results of Loeb and Walsh include the bounded energy finite solutions of the equation $\Delta u = Pu$ with $\int P < \infty$. The boundary that is relevant for these solutions in the axiomatic theory depends heavily on the choice of P. The results given below show that the harmonic part of the Royden boundary serves to determine all energy-finite solutions for any $P \geqq 0$.

We now present the precise formulation of our results. Their complete proofs and a more detailed discussion of their relation to the literature will appear in [1], [2].

Let R be a noncompact orientable Riemannian n-manifold. Consider the vector lattice $\tilde{M}$ of real-valued functions f on R which have weak exterior derivatives df with $D(f) = \int_R df \wedge {}^*df < \infty$. The bounded functions in $\tilde{M}$ form the Royden algebra M associated with R. The Royden compactification R^* of R is the compact Hausdorff space containing R as an open dense subset such that M extends continuously to R^* and also separates points of R^*.

We say that a sequence $\{f_n\}$ converges to f in the BD-topology on M if it is uniformly bounded, converges to f uniformly on compact subsets of R and $D(f-f_n) \to 0$. It is known that M is complete. The BD-closure of the functions in M with compact support is denoted by M_Δ. The compact set $\Delta = \{p \in R^* \mid f(p) = 0 \text{ for every } f \in M_\Delta\}$ is known as the harmonic part of the Royden boundary. Among the points of $R^* - R$ it is those of Δ that are significant.

[1] The authors are grateful to Professors L. Sario and M. Nakai for making available the manuscript of their forthcoming book *Classification theory*. Professor John S. M. Chang has independently obtained some of these results in the case $P \equiv 0$.