ON BOUNDARY REGULARITY FOR PLATEAU'S PROBLEM[1]

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We state here sufficient conditions for certain minimal surfaces to be differentiable at boundary points.

Let m and n be integers with $1<m<n$. We adopt the notation of [3]. See also [2]. In particular, $I_m(R^n)$ is the group of m dimensional integral currents in R^n. If $T\in I_m(R^n)$, $M(T)$ is the mass of T and ∂T is the boundary of T; if $a\in R^n$, $\Theta^m(\|T\|, a)$ is the m dimensional density of the variation measure $\|T\|$ at a.

If $T\in I_m(R^n)$, we say T is minimal if there exists $r>0$ such that $M(T)\leqq M(S+T)$ whenever $a\in R^n$, $S\in I_m(R^n)$, $\partial S=0$ and spt $S \subset\{x: |x-a|<r\}$. Given $B\in I_{m-1}(R^n)$ with $\partial B=0$, it is shown in [3] that there exists $T\in I_m(R^n)$ such that $\partial T=B$ and $M(T) \leqq M(S+T)$ whenever $S\in I_m(R^n)$ with $\partial S=0$.

THEOREM. *Suppose $T\in I_m(R^n)$, T is minimal, $a\in$ spt ∂T, $p\geqq 2$, $\Theta^{m-1}(\|\partial T\|, a)=1$ and spt ∂T intersects some neighborhood of a in a class p (real analytic) $m-1$ dimensional submanifold of R^n.*

(1) *If $\Theta^m(\|T\|, a)=1/2$, then the intersection of* spt T *with some neighborhood of a is a subset of some class $p-1$ (real analytic) m dimensional submanifold of R^n.*

(2) *If there exist independent linear functionals α_i, $i=1, \cdots, n-m+1$, on R^n such that either*

$$\operatorname{spt} \partial T \subset \{x: \alpha_i(x-a) \geqq 0,\ i=1, \cdots, n-m+1\},$$

or there is $r>0$ such that

$$\{x: |x-a| < r\} \cap \operatorname{spt} T \subset \{x: \alpha_i(x-a) \geqq 0,\ i=1, \cdots, n-m+1\},$$

then $\Theta^m(\|T\|, a)=1/2$.

COROLLARY. *Suppose $p\geqq 2$ and B is the $m-1$ dimensional integral current corresponding to some compact oriented class p (real analytic) $m-1$ dimensional submanifold N of R^n. If N lies on the boundary of some uniformly convex open subset of R^n and $T\in I_m(R^n)$ is minimal*

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