RESIDUALLY FINITE GROUPS

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1. **Introduction.** The definition of groups in terms of generators and defining relations became important when H. Poincaré discovered the fundamental (or first homotopy) group of an arcwise connected topological space. In many cases, these fundamental groups can be defined easily in terms of generators and defining relations but not otherwise in a purely group theoretical manner (i.e. without reference to the underlying space).

We shall write

$$\langle a_\sigma ; R_\lambda \rangle \tag{1.1}$$

for a group G with generators a_σ and defining relations $R_\lambda = 1$. Here σ, λ are, respectively, elements of indexing sets Σ, Λ where Σ is nonempty, and the R_λ (the "relators") are finite sequences or *words* in the a_σ, a_σ^{-1}. The unit element is denoted by 1. If Λ is empty, G is called free, and the a_σ are called a set of free generators. We shall call (1.1) a *presentation* of G and shall talk respectively of finitely generated, finitely related and finitely presented groups whenever Σ or Λ or both are finite sets.

It is rather obvious that any sets of symbols a_σ, a_σ^{-1} and words R_λ in these symbols define a group. However, it turns out to be extremely difficult to develop methods which allow one to extract information about groups given by a presentation (1.1) in a purely algebraic manner. The fundamental problems arising here were formulated and investigated by Dehn [17]. The first of these, the word problem, is the question: Which words in the generators of a group G represent the unit element? It became famous when Novikov [47], Boone [13] and Britton [15] exhibited finitely presented groups in which there is no general and effective procedure for determining whether a word in the given generators represents the unit element as a consequence of the given defining relations.

Investigating certain finitely presented groups arising from topology, Dehn [17], [18] found the available algebraic methods inadequate for this purpose and introduced geometric (including topologi-

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