# ANALYTIC SINGULAR INTEGRAL OPERATORS[1]

BY WILLIAM MARGULIES

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The following paper extends to real analytic manifolds the general theory of singular integral operators as described in [10] and [13].

The definition of an analytic singular integral operator is made in terms of the kernel of the operator. The symbol of the operator is discussed and in the case of an elliptic operator, a regularity theorem is proved.

It should be pointed out, however, that the regularity theorem is a purely local one. The question of obtaining a global inverse to an elliptic operator or more generally an operator with a prescribed symbol is still open.

**1. Definition and coordinate invariance.** We recall the definition of a modified homogeneous distribution of degree $\lambda$.

(i) If $\lambda$ is not a positive integer, then a modified homogeneous distribution of degree $\lambda$ is a homogeneous distribution of degree $\lambda$.

(ii) If $\lambda$ is a positive integer $\geqq 0$, then $f$ is a modified homogeneous distribution of degree $\lambda$ iff $f = g_\lambda + P_\lambda(x) \log |x|$, where $g_\lambda$ is orthogonal to all polynomials homogeneous of degree $\lambda$ and $P_\lambda(x)$ is a homogeneous polynomial of degree $\lambda$.

Let $M$ be a compact, real analytic manifold without boundary of dim $\nu$.

DEFINITION 1.1. $A$ is an analytic singular integral operator of order $\lambda$ iff

(i) the kernel of $A$ is analytic off the diagonal in $M \times M$;

(ii) for each $\Psi$, a coordinate function, and $p$ in the domain of $\Psi$ with $\Psi(p) = x_0$, there is an $\epsilon > 0$ such that

$$A_x = \sum_{i \geqq 0} A_x^{i-\lambda-\nu} + C_x \tag{1}$$

where $A_x^{i-\lambda-\nu}$ is a modified homogeneous distribution of degree $i - \lambda - \nu$ with a kernel

$$A_{i-\lambda-\nu}(x, z)$$

satisfying

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