ON ALMOST PERIODIC DIFFERENTIAL EQUATIONS

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Communicated by H. A. Antosiewicz, July 13, 1964

The notion of an invariant set has been widely exploited in the theory of autonomous ordinary differential equations. The purpose of this announcement is to give a natural and useful generalization of invariant sets to almost periodic systems. The proofs and several applications and extensions of our results will appear elsewhere.

We consider systems of n first order, almost periodic, ordinary differential equations of the form

$$x' = P(t, x) \qquad \left(' = \frac{d}{dt}\right), \tag{E}$$

and perturbations of the form

$$x' = P(t, x) + R(t, x) + G(t, x). \tag{PE}$$

The main result of this paper is Theorem 1 below. This rather abstract-looking theorem motivates our generalization of invariant sets. Moreover, Theorem 1 has applications to perturbation theory and to the theory of Liapunov functions. In fact, several known results for asymptotic behavior of solutions of nonlinear systems may be proved using this result (cf. [6], [7], [8], [10], and [5, p. 69]).

Let D be a fixed open domain in n-space R^n. Let Q be a fixed subset of D which is closed in the topology of D. We assume that the functions P, R, and G of systems (E) and (PE) satisfy the following hypotheses.

(H1) R and G are continuous on $I\times D$, $I=\{t;\ 0\leqq t<\infty\}$.

(H2) P is continuous on $R^1\times D$ and for each compact subset D^* of D, P is uniformly continuous on $R^1\times D^*$.

(H3) For each fixed x in D, $P(t, x)$ is almost periodic as a function of t in the sense of H. Bohr, cf. [4].

(H4) If $y(t)$ is any continuous function on the interval I with values in a compact set $D^*\subset D$, then $|G(t, y(t))|\in L_1(0, \infty)$, where $|G(t, y(t))|$ is the vector norm of $G(t; y(t))$.

(H5) $R(t, x)\to 0$ as $t\to\infty$ uniformly for x on compact subsets of Q.

(H6) For each $\epsilon>0$ and each x in Q there are numbers $T>0$ and $\delta>0$ such that whenever $t\geqq T$ and $|x-y|<\delta$ one has

[1] Partially supported by the National Science Foundation (G-24335).