ON THE REDUCTION THEORY OF VON NEUMANN[1]

BY SHÔICHIRÔ SAKAI

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1. **Introduction.** The reduction theory of von Neumann has been reformulated and modernized by many authors (cf. [1], [5]).

However, in all of them, a W^*-algebra has been considered as an operator algebra on a Hilbert space. Therefore in order to construct the reduction theory, the notion of the direct integral of Hilbert spaces has been used and it makes the theory very complicated. On the other hand, the author [6] showed that a W^*-algebra can be intrinsically characterized as a C^*-algebra with a dual structure as a Banach space; therefore it can be imagined that the reduction theory can be space-freely developed. Along this line, the author [7] gave a new approach to the reduction theory. This approach gives an exact formulation for the problem of extending the reduction theory to nonseparable cases and moreover suggests the possibility of the extension of the theory to more general Banach algebras, because of the use of general theorems in functional analysis (the Dunford-Pettis theorem [2] and Grothendieck's theorem [3]). However, the proof given in the lecture notes [7] was still complicated. In this note, we give a simple proof for the fundamental theorems in the new approach in a more general form and in addition we state some related problems.

2. First of all, we state some facts concerning the tensor product of Banach spaces. Let E and F be two Banach spaces, $E\otimes F$ the algebraic tensor product of E and F. A norm α on $E\otimes F$ is said to be a cross norm, if for every $x\in E$ and $y\in F$, $\alpha(x\otimes y)=\|x\|\,\|y\|$. $E\otimes_\alpha F$ denotes the completion of $E\otimes F$ with respect to α. The "least cross norm" λ is obtained by the natural algebraic imbedding of $E\otimes F$ into $L(E^*, F)$, where $L(E^*, F)$ is the Banach space of all bounded linear operators of E^* into F. If under this mapping, $T^u\in L(E^*, F)$ corresponds to a tensor $u=\sum_{j=1}^n x_j\otimes y_j$, then for $x^*\in E^*$

$$T^u x^* = \sum_{j=1}^n \langle x_j, x^*\rangle y_j.$$

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