BOOK REVIEWS

Functionals of finite Riemann surfaces. By M. M. Schiffer and D. C. Spencer. Princeton University Press, 1954. 10+451 pp. $8.00.

In the terminology of the authors a finite Riemann surface is a Riemann surface of finite genus with a finite number of nondegenerate boundary components. Such a surface has a double, obtained by reflection across the boundary, and one of the main features of the book is the systematic use of this symmetrization process. The authors even introduce a disconnected double of a closed Riemann surface, and for nonorientable Riemann surfaces the double is a two-sheeted orientable covering surface.

The first three chapters contain a development of the classical theory. The topological classification is taken for granted, but the existence theorems are carefully proved by the method of orthogonal projection in de Rham's and Kodaira's version. The treatment of Abelian differentials includes the Riemann-Roch theorem. A somewhat disturbing omission in this introductory part is the lack of practically all references to covering surfaces, in spite of their use later on.

The more advanced theory begins with the fourth chapter. The main interest, throughout the book, is attached to the so-called *domain functionals*, represented by Green's and Neumann's function, harmonic measure, the period matrix, and similar quantities. The first task is to compare different functionals of the same domain, and this is accomplished by deriving all functionals from a common source. The simplest way is to start from normalized integrals of the third kind.

We introduce some of the notations used by the authors. For a closed surface $\mathcal{J}$, let $\Omega_{qq_0}(p)$ be the integral with logarithmic poles at q, q_0 and single-valued real part. Then

$$\frac{\partial^2 \Omega_{qq_0}(p)}{\partial p \partial q} = -\frac{1}{[z(p) - z(q)]^2} + \text{regular terms}$$

is a bilinear differential on $\mathcal{J}$ (independent of the auxiliary point q_0). Suppose now that $\mathcal{J}$ is the double of $\mathfrak{M}$, and let $\tilde{q}$ denote the symmetric point of q. The authors introduce

$$L(p, q) = -\frac{1}{\pi} \frac{\partial^2 \Omega_{q\tilde{q}}(p)}{\partial p \partial q}$$

and prove on one hand the symmetry relations