THE STRUCTURE OF MINIMAL SETS

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1. **Introduction.** A minimal set is a topological space X acted on by a topological group T such that the orbit closure of every point in X coincides with X. If G is a relatively dense subgroup of T, the orbit closure under G of a point of X may or may not coincide with X. In this paper the properties of the orbit closures under such a subgroup are studied and several of the possibilities are analyzed. There is displayed an example of a regularly almost periodic point such that not all points in the orbit closure are regularly almost periodic.

2. **The decomposition of a minimal set by relatively dense subgroups.** Let X be a compact metric space and let T be an additive abelian topological group. Let f be a continuous transformation of the product space $X \times T$ into X. We denote the image of the point $x \times t$ under f by either $f(x, t)$ or $f^t(x)$. We assume that f defines a transformation group in that if $t, s \in T$, $x \in X$, then

$$f^0(x) = x, \tag{1}$$

$$f^{s+t}(x) = f^s(f^t(x)). \tag{2}$$

It is easily shown that for fixed t in T, the transformation defined by $x \rightarrow f^t(x)$ is a homeomorphism of X onto X. With f satisfying the stated conditions we say that T is a *transformation group* acting on X.

The subset Y of X is *invariant* under the subset A of T if $f^a(Y) = Y$ for all a in A. If G is a subgroup of T and Y is a subset of X which is invariant under G, then Y is a topological space, G is a topological group, and f defines a continuous transformation of the product $Y \times G$ onto Y such that (1) and (2) are satisfied for all points y in Y and all element pairs g, h in G. Thus G is a transformation group acting on Y.

The *orbit* of x is the set $f(x, T)$. If A is a subset of T, the *orbit of x under A* is the set $f(x, A)$.

The space X is *minimal* under T if for every x in X, $\bar{f}(x, T) = X$, where $\bar{f}(x, T)$ denotes the closure of the set $f(x, T)$. The subset A of T is *relatively dense* in T if there exists a compact subset C of T such that $T = A + C$. The point x of X is *almost periodic* under T if, corresponding to any neighborhood U of x, there exists a set A, relatively dense in T, such that $f(x, A) \subset U$.

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