let x be fixed, $-1<x<1$. We obtain for the roots of the polynomial (19) in z the condition

$$\frac{1+xz}{(1+2xz+z^2)^{1/2}}=x_\nu \tag{20}$$

where x_ν denotes a root of P_n. Or

$$z=\frac{x(x_\nu^2-1)\pm x_\nu((1-x_\nu^2)(1-x^2))^{1/2}}{x^2-x_\nu^2}, \tag{21}$$

thus the roots in z are all real. Using the trivial inequality (16) the assertion follows.

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NOTE ON THE EIGENVALUES OF THE STURM-LIOUVILLE DIFFERENTIAL EQUATION

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In discussing eigenvalues and eigenfunctions of the Sturm-Liouville differential equation

$$L(u)+\lambda\rho u=0,\qquad L(u)=(pu')'-qu,$$

with

$$\left.\begin{aligned} p(x)&\geqq m>0\\ q(x)&\geqq 0\\ \beta\geqq\rho(x)&\geqq\alpha>0\end{aligned}\right\}\quad\text{for } a\leqq x\leqq b,\text{ and for some } \alpha,\ \beta,\text{ and } m,$$

and the boundary conditions

$$u(a)=c_1u(b),\qquad u'(a)=c_2u'(b),\qquad c_1c_2p(a)=p(b),$$

we find that we can represent our eigenfunctions as unit normals in the directions of the principal axes of an ellipsoid in function space. We define our function space F as the set of all functions $v(x)$, $a\leqq x\leqq b$, which satisfy the boundary conditions of the Sturm-Liouville equation. The origin of our space will be the function $u(x)=0$. We can now metrize F by defining our inner product $(u,\ v)$ for

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