THE CENTER OF A JORDAN RING[1]

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If $\mathfrak{A}$ is an arbitrary associative ring we can symmetrize and anti-symmetrize the multiplication defined in $\mathfrak{A}$ to obtain two non-associative rings. We set

$$\{ab\} = ab + ba, \qquad [ab] = ab - ba \tag{1}$$

and call the former the *Jordan product* and the latter the *commutator* or *Lie product* of a and b. If we use $\{ab\}$ as product in place of the originally defined ab we obtain the *Jordan ring* $\mathfrak{A}_j$ determined by $\mathfrak{A}$. Similarly the *Lie ring* $\mathfrak{A}_l$ is obtained by using $[ab]$ in place of ab. Naturally if $\mathfrak{A}$ has characteristic 2 then $\mathfrak{A}_j = \mathfrak{A}_l$. It is customary to exclude this case from consideration but in most of our discussion we shall not find it necessary to do so. Clearly $\{ab\} = \{ba\}$, $[ab] = -[ba]$. Also we recall the following well known identity of Jacobi's:

$$[[ab]c] + [[bc]a] + [[ca]b] = 0. \tag{2}$$

If $\mathfrak{R}$ is any non-associative ring one defines the center of $\mathfrak{R}$ to be the totality of elements c that commute,

$$c \cdot a = a \cdot c, \tag{3}$$

and associate,

$$\begin{gathered} (a \cdot b) \cdot c = a \cdot (b \cdot c), \qquad (a \cdot c) \cdot b = a \cdot (c \cdot b), \\ (c \cdot a) \cdot b = c \cdot (a \cdot b), \end{gathered} \tag{4}$$

with all a, b in $\mathfrak{R}$.[2] It is known that the center is a subring of $\mathfrak{R}$. Clearly this subring is associative. It is also known that the center of a simple ring is either 0 or a field. It is easy to see that the middle condition in (4) is a consequence of (3) and the other conditions in (4). Also it is clear that if $\mathfrak{R}$ is commutative then the first condition of (4) characterizes the center.

We consider now the centers $\mathfrak{C}_j$ and $\mathfrak{C}_l$ respectively of $\mathfrak{A}_j$ and $\mathfrak{A}_l$.

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[2] See Jacobson, *Structure theory of simple rings without finiteness assumptions*, Trans. Amer. Math. Soc. vol. 57 (1945) p. 239, or T. Nakayama, *Über einfache distributive Systeme unendlicher Ränge*, Proc. Imp. Acad. Tokyo vol. 20 (1944) p. 62 for this definition and for the results quoted in this paragraph.