ON THE ZEROS OF POLYNOMIALS WITH COMPLEX COEFFICIENTS[1]

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1. **Introduction.** Problems in dynamics very frequently have physically realizable solutions only if the determinantal equation of the system has all its roots in the negative half of the complex plane. It is therefore convenient to have a simple algorithm for testing whether this condition holds without actually computing the roots. Solutions to this problem have been considered by Cauchy [1],[2] Routh [6], and many others. Hurwitz [4] gave a method for polynomials with *real* coefficients of the form

$$(1.1) \qquad P(z) = z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n.$$

According to his rule, all of the roots lie in the half-plane $R(z) < 0$ if and only if all the determinants

$$D_p = \begin{vmatrix} a_1, & a_3, & a_5, & \cdots, & a_{2p-1} \\ 1, & a_2, & a_4, & \cdots, & a_{2p-2} \\ 0, & a_1, & a_3, & \cdots, & a_{2p-3} \\ 0, & 1, & a_2, & \cdots, & a_{2p-4} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_p \end{vmatrix}, \qquad p = 1, 2, \cdots, n, \; a_j = 0, \; j > n,$$

are positive.

Recently, Wall [8] formulated and proved this theorem by means of continued fractions. We extend his method to polynomials with *complex* coefficients,

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The referee has kindly called my attention to a recent article by Herbert Bilharz, *Bemerkung zu einem Satze von Hurwitz*, Zeitschrift für Angewandte Mathematik und Mechanik vol. 24 (1944) pp. 77–82 (lithoprinted by Edwards Brothers, Inc., Ann Arbor, Mich., 1945). There is presented in Bilharz' article an algorithm for the computation of determinants of type D_p similar to that given here in §2. Also Theorem 3.2 is essentially the same as the theorem stated and proved by Bilharz (p. 81), and Theorem 4.1 is equivalent to but approached differently from that stated by Bilharz without detailed proof.

[2] Numbers in brackets refer to the bibliography at the end of the paper.